Maximum Wave Checks

Most wave theories (both analytic and numerically based) are capable of yielding valid mathematical solutions to physically implausible data; particularly with regards to wave steepness and depth-related breaking. In part, this is a consequence of some of the assumptions imposed upon the boundary problem formulation. As an aid in restricting solutions to an observed physically valid domain, empirical data and formulations are often employed to estimate the validity of the given wave. The following expression (Fenton, 1990) is used for estimating the greatest wave as a function of both wavelength and depth:

$$H_{max} = d\left\{ \frac{.141063\frac{L}{d} + .0095721\left(\frac{L}{d}\right)^2 + .0077829\left(\frac{L}{d}\right)^3}{1 + .078834\frac{L}{d} + .0317567\left(\frac{L}{d}\right)^2 + .0093407\left(\frac{L}{d}\right)^3} \right\} \tag{15}$$

In the limits, the leading term in the numerator of the above expression provides the familiar steepness limit for short waves ($H_{max}/L \rightarrow .141063$), and as ($\lim L/d \rightarrow \infty$) the ratio of coefficients of the cubic terms provides the familiar ratio ($.0077829(L/d)^3/.0093407(L/d)^3 \rightarrow .83322$). This simple empirical test is applied using the given water depth, and solved wavelength as a rough filter for implausible wave specifications.

Derived Results

Traditional engineering quantities of interest about the wave are derived from the solution of the governing equation. Since the solution is expressed as a Fourier series, many of the derived quantities will also be functions of the series. Formulas for kinematics, integral properties, and other relevant items are included in the following tabulations. All quantities are relative to the stationary (non-moving) frame of reference.

Kinematics and Other Derived Variables

Velocities:

Horizontal: $$u(x, z) = \frac{\partial \psi}{\partial z} = -\bar{u} + \left(\frac{g}{k}\right)^{1/2} \sum_{j=1}^{N} jB_j \frac{\cosh jk(d+z)}{\cosh jkd} \cos jkx \tag{16}$$

Vertical: $$w(x, z) = -\frac{\partial \psi}{\partial x} = \left(\frac{g}{k}\right)^{1/2} \sum_{j=1}^{N} jB_j \frac{\sinh jk(d+z)}{\cosh jkd} \sin jkx \tag{17}$$

Accelerations:

Horizontal: $$a_x(x, y) = \frac{du}{dt} = u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} \tag{18}$$

Vertical: $$a_z(x, y) = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = u\frac{\partial u}{\partial z} - w\frac{\partial u}{\partial x} \tag{19}$$

where

$$\frac{\partial u}{\partial x} = -(gk)^{1/2} \sum_{j=1}^{N} j^2 B_j \frac{\cosh jk(d+z)}{\cosh jkd} \sin jkx$$

$$\frac{\partial u}{\partial z} = (gk)^{1/2} \sum_{j=1}^{N} j^2 B_j \frac{\sinh jk(d+z)}{\cosh jkd} \cos jkx$$